Free vibrations of steel pole with variable cross-section of arbitrary shape

Static scheme

Cantilever with length -

Material

Steel

Elastic modulus - Poisson′s ratio -

Shear modulus -

Mass density -

Cross-section

Section height:

- at bottom -

- at top -

- difference -

- as a function of distance to base -

Section width:

- at bottom -

- at top -

- difference -

- as a function of distance to base -

Cross-section shape - **Elliptical tube**

Thickness - , ,

Function of cross-section outline

Cross section properties

Area -

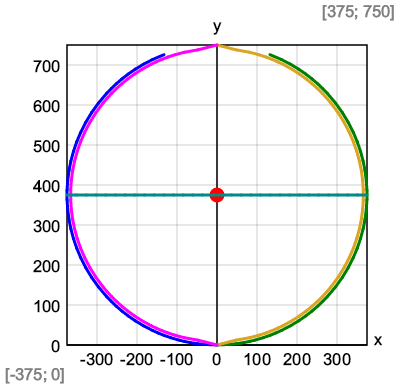
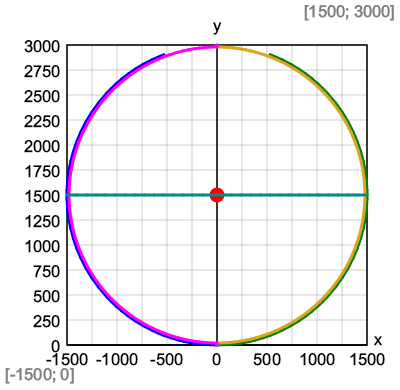
First moment of area -

Geometrical center-

Second moment of area

First moment of area under z

Shear area



Mass

Distributed mass along height -

Solution

Number of nodes -

Length of one segment -

Elevation of node *j* -

Bending due to horizontal force *F*j = 1 at node *j* -

Flexibility matrix

Mass matrix

Total mass of structure - t

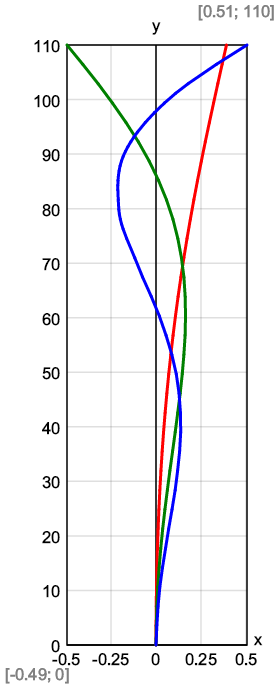
Eigenvalues

Natural frequencies

Vibration frequencies

Vibration periods

Mode shapes



Comparison with ASCE SEI 7/22 (C26.11-13)

Fundamental natural frequency (C26.11-12)

Fundamental period of vibrations

Comparison with SAP 2000 structural analysis software

Input data:

**S T A T I C L O A D C A S E S**

STATIC CASE SELF WT

CASE TYPE FACTOR

LOAD1 DEAD 0.0000

**J O I N T D A T A**

JOINT GLOBAL-X GLOBAL-Y GLOBAL-Z RESTRAINTS ANGLE-A ANGLE-B ANGLE-C

1 0.00000 0.00000 0.00000 1 1 1 1 1 1 0.000 0.000 0.000

2 0.00000 0.00000 10.00000 0 0 0 0 0 0 0.000 0.000 0.000

3 0.00000 0.00000 20.00000 0 0 0 0 0 0 0.000 0.000 0.000

4 0.00000 0.00000 30.00000 0 0 0 0 0 0 0.000 0.000 0.000

5 0.00000 0.00000 40.00000 0 0 0 0 0 0 0.000 0.000 0.000

6 0.00000 0.00000 50.00000 0 0 0 0 0 0 0.000 0.000 0.000

7 0.00000 0.00000 60.00000 0 0 0 0 0 0 0.000 0.000 0.000

8 0.00000 0.00000 70.00000 0 0 0 0 0 0 0.000 0.000 0.000

9 0.00000 0.00000 80.00000 0 0 0 0 0 0 0.000 0.000 0.000

10 0.00000 0.00000 90.00000 0 0 0 0 0 0 0.000 0.000 0.000

11 0.00000 0.00000 100.00000 0 0 0 0 0 0 0.000 0.000 0.000

12 0.00000 0.00000 110.00000 0 0 0 0 0 0 0.000 0.000 0.000

**F R A M E E L E M E N T D A T A**

FRAME JNT-1 JNT-2 SECTION ANGLE RELEASES SEGMENTS R1 R2 FACTOR LENGTH

13 1 2 VAR01 0.000 000000 2 0.000 0.000 1.000 10.000

14 2 3 VAR02 0.000 000000 2 0.000 0.000 1.000 10.000

15 3 4 VAR03 0.000 000000 2 0.000 0.000 1.000 10.000

16 4 5 VAR04 0.000 000000 2 0.000 0.000 1.000 10.000

17 5 6 VAR05 0.000 000000 2 0.000 0.000 1.000 10.000

18 6 7 VAR06 0.000 000000 2 0.000 0.000 1.000 10.000

19 7 8 VAR07 0.000 000000 2 0.000 0.000 1.000 10.000

20 8 9 VAR08 0.000 000000 2 0.000 0.000 1.000 10.000

21 9 10 VAR09 0.000 000000 2 0.000 0.000 1.000 10.000

22 10 11 VAR10 0.000 000000 2 0.000 0.000 1.000 10.000

23 11 12 VAR11 0.000 000000 2 0.000 0.000 1.000 10.000

**M A T E R I A L P R O P E R T Y D A T A**

MAT MODULUS OF POISSON'S THERMAL WEIGHT PER MASS PER

LABEL ELASTICITY RATIO COEFF UNIT VOL UNIT VOL

STEEL 206000000 0.300 1.120E-05 78.500 7.850

CONC 24821128.4 0.200 9.900E-06 23.562 2.401

OTHER 24821128.4 0.200 9.900E-06 23.562 2.401

**F R A M E S E C T I O N P R O P E R T Y D A T A**

SECTION MAT SECTION DEPTH FLANGE FLANGE WEB FLANGE FLANGE

LABEL LABEL TYPE WIDTH THICK THICK WIDTH THICK

TOP TOP BOTTOM BOTTOM

P00 STEEL 3.000 0.000 0.000 2.000E-02 0.000 0.000

P01 STEEL 2.795 0.000 0.000 1.900E-02 0.000 0.000

P02 STEEL 2.591 0.000 0.000 1.800E-02 0.000 0.000

P03 STEEL 2.386 0.000 0.000 1.700E-02 0.000 0.000

P04 STEEL 2.182 0.000 0.000 1.600E-02 0.000 0.000

P05 STEEL 1.977 0.000 0.000 1.500E-02 0.000 0.000

P06 STEEL 1.773 0.000 0.000 1.400E-02 0.000 0.000

P07 STEEL 1.568 0.000 0.000 1.300E-02 0.000 0.000

P08 STEEL 1.364 0.000 0.000 1.200E-02 0.000 0.000

P09 STEEL 1.159 0.000 0.000 1.100E-02 0.000 0.000

P10 STEEL 0.955 0.000 0.000 1.000E-02 0.000 0.000

P11 STEEL 0.750 0.000 0.000 9.000E-03 0.000 0.000

**F R A M E S E C T I O N P R O P E R T Y D A T A**

SECTION AREA TORSIONAL MOMENTS OF INERTIA SHEAR AREAS

LABEL INERTIA I33 I22 A2 A3

P00 0.187 0.416 0.208 0.208 9.462E-02 9.462E-02

P01 0.166 0.319 0.160 0.160 8.375E-02 8.375E-02

P02 0.145 0.241 0.120 0.120 7.356E-02 7.356E-02

P03 0.127 0.178 8.876E-02 8.876E-02 6.398E-02 6.398E-02

P04 0.109 0.128 6.385E-02 6.385E-02 5.508E-02 5.508E-02

P05 9.246E-02 8.898E-02 4.449E-02 4.449E-02 4.679E-02 4.679E-02

P06 7.736E-02 5.985E-02 2.992E-02 2.992E-02 3.917E-02 3.917E-02

P07 6.351E-02 3.839E-02 1.920E-02 1.920E-02 3.217E-02 3.217E-02

P08 5.097E-02 2.329E-02 1.165E-02 1.165E-02 2.584E-02 2.584E-02

P09 3.967E-02 1.307E-02 6.536E-03 6.536E-03 2.014E-02 2.014E-02

P10 2.969E-02 6.629E-03 3.314E-03 3.314E-03 1.509E-02 1.509E-02

P11 2.095E-02 2.876E-03 1.438E-03 1.438E-03 1.068E-02 1.068E-02

**F R A M E S E C T I O N P R O P E R T Y D A T A**

SECTION SECTION MODULII PLASTIC MODULII RADII OF GYRATION

LABEL S33 S22 Z33 Z22 R33 R22

P00 0.139 0.139 0.178 0.178 1.054 1.054

P01 0.114 0.114 0.146 0.146 0.981 0.981

P02 9.295E-02 9.295E-02 0.119 0.119 0.910 0.910

P03 7.440E-02 7.440E-02 9.541E-02 9.541E-02 0.838 0.838

P04 5.853E-02 5.853E-02 7.507E-02 7.507E-02 0.766 0.766

P05 4.501E-02 4.501E-02 5.774E-02 5.774E-02 0.694 0.694

P06 3.375E-02 3.375E-02 4.332E-02 4.332E-02 0.622 0.622

P07 2.449E-02 2.449E-02 3.144E-02 3.144E-02 0.550 0.550

P08 1.708E-02 1.708E-02 2.194E-02 2.194E-02 0.478 0.478

P09 1.128E-02 1.128E-02 1.450E-02 1.450E-02 0.406 0.406

P10 6.941E-03 6.941E-03 8.931E-03 8.931E-03 0.334 0.334

P11 3.835E-03 3.835E-03 4.942E-03 4.942E-03 0.262 0.262

Results:

**A S S E M B L E D J O I N T M A S S E S IN GLOBAL COORDINATES**

JOINT UX UY UZ RX RY RZ

1 7.067329 7.067329 7.067329 .000000 .000000 .000000

2 13.024974 13.024974 13.024974 .000000 .000000 .000000

3 11.437717 11.437717 11.437717 .000000 .000000 .000000

4 9.949356 9.949356 9.949356 .000000 .000000 .000000

5 8.562760 8.562760 8.562760 .000000 .000000 .000000

6 7.275225 7.275225 7.275225 .000000 .000000 .000000

7 6.089295 6.089295 6.089295 .000000 .000000 .000000

8 5.002586 5.002586 5.002586 .000000 .000000 .000000

9 4.017318 4.017318 4.017318 .000000 .000000 .000000

10 3.131435 3.131435 3.131435 .000000 .000000 .000000

11 2.346830 2.346830 2.346830 .000000 .000000 .000000

12 0.936644 0.936644 0.936644 .000000 .000000 .000000

**T O T A L A S S E M B L E D J O I N T M A S S E S IN GLOBAL COORDINATES**

UX UY UZ RX RY RZ

TOTAL 78.841467 78.841467 78.841467 .000000 .000000 .000000

**M O D A L P E R I O D S A N D F R E Q U E N C I E S**

MODE PERIOD FREQUENCY FREQUENCY EIGENVALUE  
 (TIME) (CYC/TIME) (RAD/TIME) (RAD/TIME)\*\*2

1 2.784527 0.359127 2.256464 5.091629

2 0.831336 1.202883 7.557935 57.122388

3 0.369511 2.706277 17.004037 289.137267

Modal shapes

Картина, която съдържа линия, диаграма, Детско изкуство

Описанието е генерирано автоматично

Ф1 Ф2 Ф3

Differences:

First mode - *δ*1 = (2.773 – 2.785)/2.773 = 0.43%

Second mode - *δ*2 = (0.826 – 0.831)/0.826 = 0.60%

Third mode - *δ*3 = (0.367 – 0.370)/0.367 = 0.81%

Convergence of solution in dependance of mesh density:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Number of nodes | Period, s | | | Difference, % | | |
| Ф1 | Ф2 | Ф3 | Ф1 | Ф2 | Ф3 |
| 3 | 3,02 | 1,06 | 0,440 | 9,42% | 30,86% | 24,65% |
| 6 | 2,82 | 0,870 | 0,401 | 2,17% | 7,41% | 13,60% |
| 12 | 2,77 | 0,824 | 0,365 | 0,36% | 1,73% | 3,40% |
| 24 | 2,76 | 0,813 | 0,356 | 0,00% | 0,37% | 0,85% |
| 48 | 2,76 | 0,810 | 0,353 | 0,00% | 0,00% | 0,00% |
| 96 | 2,76 | 0,810 | 0,353 | 0,00% | 0,00% | 0,00% |
| SAP (11) | 2,79 | 0,831 | 0,370 | 0,91% | 2,59% | 4,82% |

Plot of convergence vs number of nodes:

Period Difference