Third order geometric nonlinearity analysis of a double-bar Biot truss

(solved by four different numerical methods)

Input data

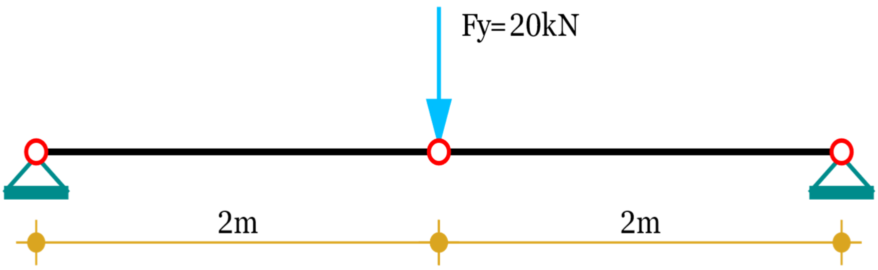
Strut length - $l = 2\,\mathrm{m}$

Material - steel. Modulus of elasticity - $E = 210\,\mathrm{GPa}$

Cross section - circular with diameter $\Phi = 20\,\mathrm{mm}$. Area - $A = \frac{\pi \cdot \Phi^2}{4} = 3.14\,\mathrm{cm}^2$

Axial stiffness - $EA = E \cdot A = 65973.4\,\mathrm{kN}$

Vertical force - $F_y = 20\,\mathrm{kN}$



Solution

Because of the symmetry, the horizontal displacement in the middle is $u = 0\,\mathrm{m}$.

The vertical displacement is the only unknown - v = ? P

Since the system is linearly unstable, we use 3-rd order geometric nonlinearity theory for the solution. The equilibrium equations are then derived for the deformed state of the structure, as follows:

Length and elongation in deformed state

$l'(v) = \sqrt{(l+u)^2 + v^2}$, $\Delta l(v) = l'(v) - l$

Horizontal reaction - $F_x(v) = EA \cdot \frac{\Delta l(v)}{l} \cdot \frac{l+u}{l\prime(v)}$

Vertical reaction - $F_y(v) = EA \cdot \frac{\Delta l(v)}{l} \cdot \frac{v}{l\prime(v)}$

Vertical reaction derivative - $F'_{yv}(v) = EA \cdot \left(\frac{1}{l} - \frac{(l+u)^2}{l\prime(v)^3}\right)$

1. Fixed point iteration method

Relative strain - $\varepsilon = \frac{F_y}{2 \cdot EA} = 0.000152$

Relative precision - $\delta_{max} = 10^{-4} = 0.0001$

Initial value - $v_0 = 200\,\mathrm{mm}$

We express the unknown vertical displacement at the middle joint as a function of the vertical force:

$$v = \sqrt{\frac{1}{\left(\frac{1}{l} - \frac{\varepsilon}{v_0}\right)^2} - (l + u)^2} = 110.24\,\mathrm{mm}$$

After calculating the above expression iteratively $n = 13$ times, we get:

$v = 134.51\,\mathrm{mm}$

Relative error - $\delta = \frac{|v - v_0|}{|v|} = 7.6\times10^{-5}$

2. Newton-Raphson's method

Initial value - $v_0 = 200\,\mathrm{mm}$

We repeatedly calculate the following expression:

$$v = v_0 - \frac{2 \cdot F_y(v_0) - F_y}{F'_{yv}(v_0)} = 106.93\,\mathrm{mm}$$

After $n = 4$ iterations we get: $v = 134.51\,\mathrm{mm}$

Relative error - $\delta = \frac{|v - v_0|}{|v|} = 9\times10^{-6}$

3. Secant method

Slope reduction factor - $\alpha = 1$

Initial value - $v_0 = 200\,\mathrm{mm}$

Force value - $F_{y0} = 2 \cdot F_y(v_0) = 65.48\,\mathrm{kN}$

We calculate the first approximation using Newton-Raphson's method

$$v_1 = v_0 - \alpha \cdot \frac{F_{y0} - F_y}{2 \cdot F'_{yv}(v_0)} = 153.46\,\mathrm{mm}$$

Force value - $F_{y1} = 2 \cdot F_y(v_1) = 29.67\,\mathrm{kN}$

The next approximation is evaluated by the formula:

$$v_2 = v_1 - \alpha \cdot \left(F_{y1} - F_y\right) \cdot \frac{v_1 - v_0}{F_{y1} - F_{y0}} = 140.89\,\mathrm{mm}$$

We continue the calculations iteratively until we reach convergence.

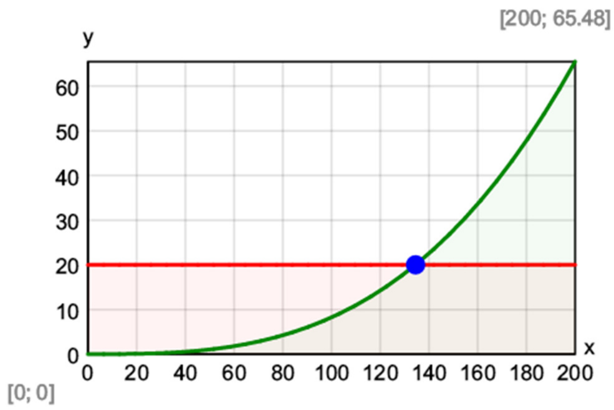
After $n = 4$ iterations we get: $v_2 = 134.51\,\mathrm{mm}$

Relative error - $\delta = \frac{|v_2 - v_1|}{|v_2|} = 1.57\times10^{-6}$

4. Solution with Calcpad (modified Anderson-Bjork's method)

$v = \$Root\{2 \cdot F_y(v) = F_y; v \in [0\,\text{mm}; 200\,\text{m}]\} = 134.51\,\text{mm}$

System behavior graph (force-displacement)



Results

Axial forces in bars - $N = \frac{\Delta l(v)}{l} \cdot EA = 149.03\,\text{kN}$

Rotation angle - $\alpha = \mathbf{atan2}(l; v) = 3.85\,°$

Reactions in supports

Horizontal - $R_x = F_x(v) = 148.69\,\text{kN} = N \cdot \mathbf{cos}(\alpha) = 148.69\,\text{kN}$

Vertical - $R_y = F_y(v) = 10\,\text{kN} = N \cdot \mathbf{sin}(\alpha) = 10\,\text{kN}$